Assignment 11

Coverage: 16.7 in Text.

Exercises: 16.7 no 3, 6, 8, 15, 17, 25.

Hand in 16.7 no 6, 15, 25 by April 20.

Supplementary Problems

1. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$. Establish Lagrange identity
$$\sum_{1 \le i < j \le n} (a_i b_j - a_j b_i)^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2 \ .$$

2. Deduce from (1) the identity
$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta \ ,$$
where θ is the angle between 3-vectors $\mathbf{a}$ and $\mathbf{b}$.

3. A regular parametrization $\mathbf{r}$ from the square $[0,1]^2$ to $S \subset \mathbb{R}^3$ is called a tube if (a) it is bijective on $(0,1)^2$ and (b) $\mathbf{r}((0,y)) = \mathbf{r}((1,y)),\ y \in [0,1]$. Show that for any irrotational C^1-vector field $\mathbf{F}$,
$$\oint_{C_1} \mathbf{F} \cdot d\mathbf{r} = \oint_{C_2} \mathbf{F} \cdot d\mathbf{r} \ ,$$
where $C_1 : x \mapsto \mathbf{r}(x, 0)$ and $C_2 : x \mapsto \mathbf{r}(x, 1)$ for $x \in [0, 1]$.

4. (Optional) Let $\mathbf{r} : D \to S$ be a regular parametrization of S so that $\mathbf{r}_u \times \mathbf{r}_v$ is the chosen normal direction of S. Let $\gamma(t)$ be a parametrization of the boundary of D in anticlockwise direction. Show that the curve $\mathbf{r}(\gamma(t))$ described the boundary of S in the orientation induced by the chosen normal of S.

Exercises 16.7

Using Stokes' Theorem to Find Line Integrals

In Exercises 1–6, use the surface integral in Stokes' Theorem to calculate the circulation of the field $\mathbf{F}$ around the curve C in the indicated direction.

6. $\mathbf{F} = x^2y^3\mathbf{i} + \mathbf{j} + z\mathbf{k}$

C: The intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$, counterclockwise when viewed from above

Stokes' Theorem for Parametrized Surfaces

In Exercises 13–18, use the surface integral in Stokes' Theorem to calculate the flux of the curl of the field $\mathbf{F}$ across the surface S in the direction of the outward unit normal $\mathbf{n}$.

15. $\mathbf{F} = x^2y\mathbf{i} + 2y^3z\mathbf{j} + 3z\mathbf{k}$

S: $\mathbf{r}(r, \theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + r\mathbf{k}$, $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$

Theory and Examples

25. Find a vector field with twice-differentiable components whose curl is $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ or prove that no such field exists.